Appendix G.

Random longitudinal velocity (RLV) line broadening and frequency dispersion.

Theoretical treatment.

Consider a wave train propagating in a medium in random motion.

Consider those components of the random motion that lie in the same ("longitudinal") direction as that of the propagation of the wave train. Assume the velocities of random motion are very small compared with the velocity (c) of wave propagation; thus the velocity structure of the medium does not change significantly during the passage of the train.

At time $t = 0$ let the local longitudinal velocity of the medium at the front of the wave train be zero relative to the overall volume of the medium under consideration (i.e. the reference frame).

Let the front of the train then enter a zone where the longitudinal velocity of the medium is increasing by b cm/s per cm travelled in the longitudinal direction. The longitudinal acceleration of the front of the train, relative to the reference frame and to that part of the train now passing where the front of the train was when $t = 0$, is then bc cm/s^2.

After t seconds the front of the train has travelled an extra $\frac{1}{2}bct^2$ cm relative to that part of the train now passing the origin.

Thus the train has been stretched by an amount $\frac{1}{2}bct^2$ in a distance ct.

The proportionate change in length of the affected part of the wave train in travelling a short distance ct_1 will therefore be

$$\frac{1}{2}bt_1 \quad \ldots\ldots\ldots\ldots \quad (G_1)$$

In this expression b, the longitudinal velocity gradient, is clearly proportional to the amplitude (e.g. the r.m.s. value) of the random motion velocities and to their (e.g. r.m.s.) spatial frequency along the propagation path.

Similarly, ct_1 might be taken as the r.m.s. spatial extent of a particular value of b, and its corresponding wave-stretching action, before the wave train moves into a region with a distinctly different value of b.

Successive regions of this kind may be called influence cells, each with the ability to produce an r.m.s. amount of stretching corresponding only to the spatial and velocity parameters of the medium.

But b is actually a random quantity with both positive and negative values, so the r.m.s. wavelength dispersion (fractional) will grow in proportion to the square root of the number (n) of repetitions of the process along the path. So the r.m.s. dispersion rate (Δ) per unit distance will be

$$\Delta = (\text{r.m.s. dispersion per influence cell}) \times (\text{no. of influence cells per unit distance})^{1/2}$$
$$= \delta\Delta \,.\, \sqrt{n}$$

where $\delta\Delta = \frac{1}{2}\,\underline{\boldsymbol{b}}t_1$

in which $\underline{\boldsymbol{b}}$ is now the r.m.s. longitudinal velocity gradient in each influence cell of the medium and t_1 is now the time taken by a wave front to traverse an influence cell.

Let the propagation space be a gas. According to continuum theory the aether between the gas particles is in random motion that is a consequence only of the particle random motions.

In the absence, at present, of direct information as to how many particles contribute an effective velocity influence to the resultant velocity of the medium (aether) at a particular point in the gas, it is not possible to determine the influence cell size directly from the gas conditions.

It is possible and useful, however, to consider how changes in the gas conditions will affect the wavelength dispersion per unit distance.

Using reasoning given in Appendix A we get the r.m.s. dispersion rate per unit distance

$\Delta \propto \underline{\boldsymbol{b}} \,.\, t_1 \,.\, \sqrt{n}$ where m = particle mass

$\propto \frac{1}{P}\sqrt{\frac{T}{m}} \,.\, P \,.\, \sqrt{\frac{1}{P}}$ T = absolute temperature

$\propto \sqrt{\frac{T}{mP}}$ P^{-3} = particle number density

$\propto \sqrt{\frac{T}{m}}$. (particle number density)$^{\frac{1}{6}}$ . . . . (G_2)

Hence the **variance** ($\Delta^2 D$) of the RLV dispersion not only grows in direct proportion to distance (D) travelled in the gas, as does the RTV redshift (RD) (see Appendix A), but depends on the same gas parameters also.

Similarly, because the dispersion mechanism actually involves compound growth of the variance (in wavelength terms) with distance, this result is strictly applicable only to unit distance or, in practice, to distances over which the resulting dispersion is still only a very small fraction of the initial wavelength. For bigger distances the wavelength variance would be given by

$$\lambda\sigma_D^2 = \lambda\Delta^2(e^{\Delta^2 D} - 1) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (G_3)$$

where σ_D is the standard deviation of the resulting dispersion and a direct measure of the spectral line-broadening.

Relationship between RLV line broadening and RTV redshift.

A comparison of Equations A_2 (Appendix A) and G_1 shows that, per influence cell, the basic mean redshift is $\frac{1}{6}\underline{\boldsymbol{a}}^2 t_1^2$ and the basic r.m.s. dispersion is $\frac{1}{2}\underline{\boldsymbol{b}} t_1$ (therefore the basic dispersion variance is $\frac{1}{4}\underline{\boldsymbol{b}}^2 t_1^2$.).

Apart from the numerical factor it is likely (though I cannot prove it) that for given gas particle velocity statistics the most probable shear velocity gradient ($\underline{\boldsymbol{a}}$) bears a simple numerical relation (which I will denote by $\underline{\boldsymbol{b}} = k\underline{\boldsymbol{a}}$) to the r.m.s. longitudinal velocity gradient ($\underline{\boldsymbol{b}}$). Consequently the ratio of the two effects is simply

$$\frac{R}{\Delta^2} = \frac{2}{3k^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(G_4) which is a constant.

Both subsequently grow exponentially with distance when the buildup of the effect has become substantial. Observationally, however, interest lies in the r.m.s. line width, not the variance, so redshift will progressively outstrip line broadening the more the processes are repeated. This will make the redshift progressively easier to observe, despite the line broadening.

Four general conclusions can be drawn.

1) The line broadening will be most likely to swamp the associated redshift under conditions of short path length and large influence cell size. This implies low particle number density and probably requires high particle velocities (high temperature) to make the effects big enough to be observable anyway. If the basic line breadth is already large, however, due to high source temperature or stellar rotation, detection of the redshift (and hence determining the RLV line-broadening present) might become a problem.

2) Under cosmological path length conditions the dominant feature becomes the very large number of influence cells along the path, notwithstanding that those cells are the largest in existence. Consequently, the redshift will emerge more and more clearly from the line breadth as the RTV redshift increases. But there is no need for all that RTV redshift to have had an intergalactic path origin. For example, given that virtually all the redshift of a quasar can be attributed to RTV redshift (a combination of cosmic and intrinsic, with only a relatively very small intrinsic velocity contribution), it should prove possible to deduce its emission temperature by deducting from the emission line breadth an allowance for total RLV broadening based on the RTV redshift. **N.B. (4 September 1996)** If, as now seems more likely, the major part of the quasar redshift is intrinsic and of AR redshift origin, with the emission line-breadth being of

primarily rotational origin, and the object distance much reduced, the foregoing argument will be less clear-cut.

3) In the case of absorption lines, the RLV dispersion acts upon the continuum, not upon the 'absence of radiation' represented by the line. The effect of this will be that the line is pinched and shallowed in a manner somewhat analogous to that suffered by an absorption line pinched between two broad emission lines.

4) If, in a particular case, the redshift of an absorption line can reasonably be attributed to RTV redshift (or if the amount attributable to it can be independently estimated) then it may be possible to estimate the temperature (or, more precisely, the particle velocity magnitude) of the absorber by subtracting from the line width an RLV dispersion allowance based on the RTV redshift.

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